Network Analysis

- Used when tasks of a project are given all with their time requirement and we have to optimize in a way that least time is taken
- Network consist of a set of points and lines connecting each other to form a path
- Events are points connected by lines
- Line connecting events is called Activity

Predecessor

Activity which must be complete before other activity start

Successor

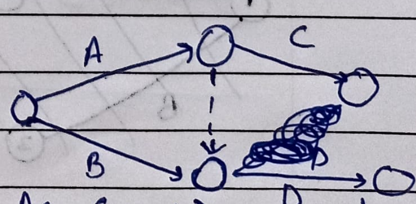
Activity which immediately after one or more of other activity is completed.

Dummy

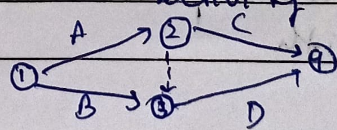
Activity which doesnot consume any resources

Q

Activity	Immediate Predecessor
A	
B	
C	A
D	A, B



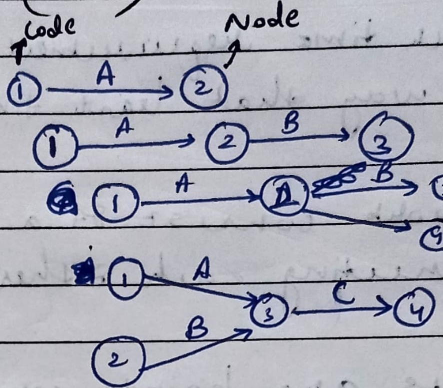
As C and D are end activity



<u>Construction of AOA Network Diagram</u>

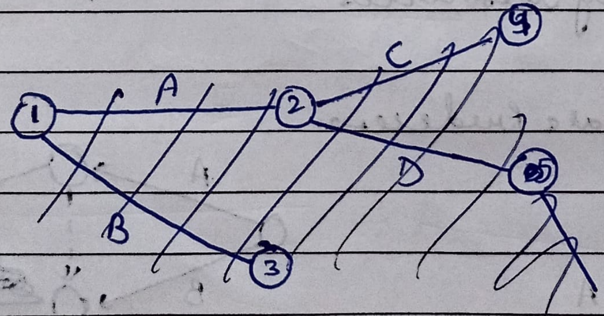
<u>Action on Arrow (AOA)</u>

1. Activity A
2. B must follow A
3. B & C must follow A
4. C must follow A, B



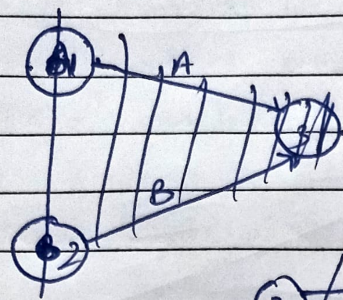
Q

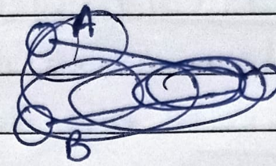
Activity	Immediate Predecessor
A	–
B	–
C	A
D	A
E	I, J, K
F	B, D
G	B, D
H	F
I	A
J	G, H
K	F

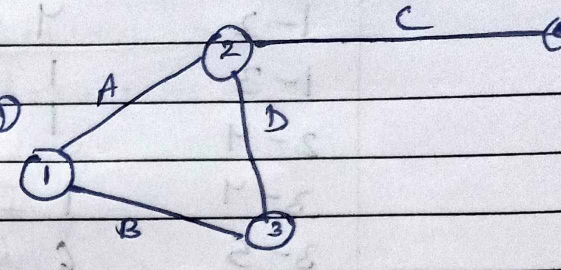


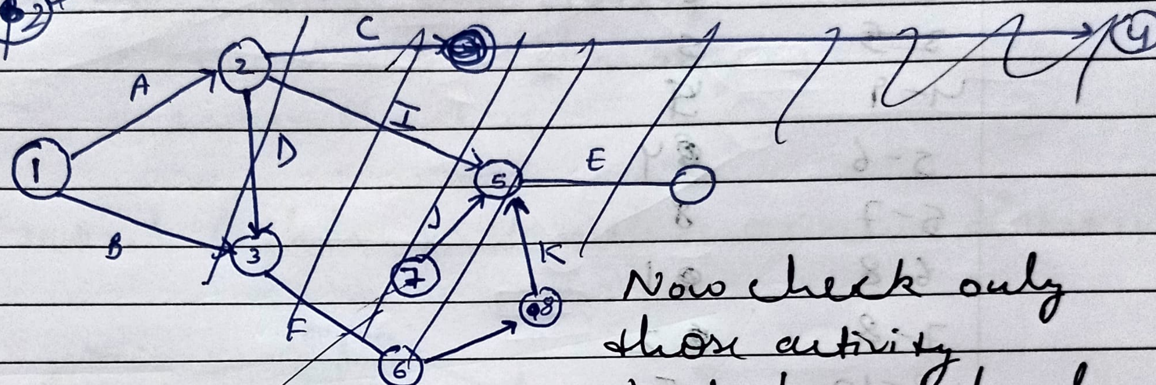
Check pair and see they are merging with

Take initial one's which have predecessor

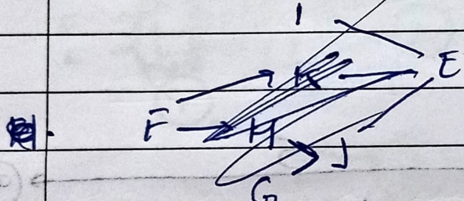




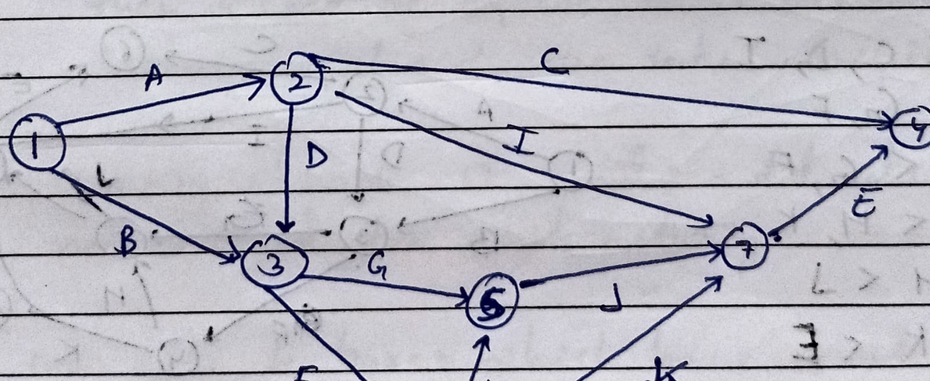




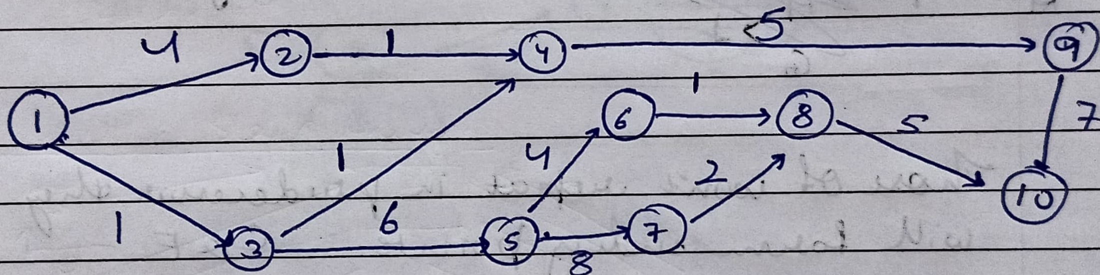
Now check only those activity which have already existing activity



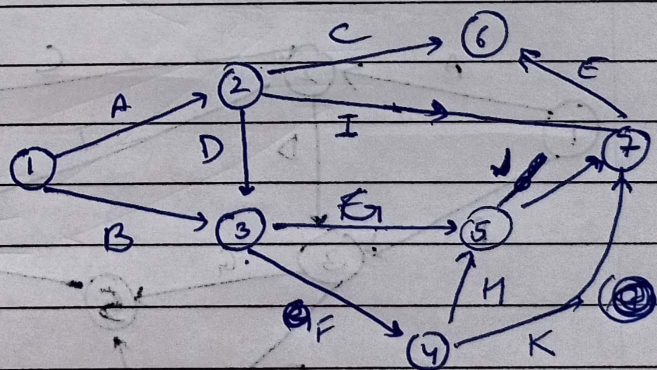
Those of won't repeat in predecessor they will form ending points : C & E

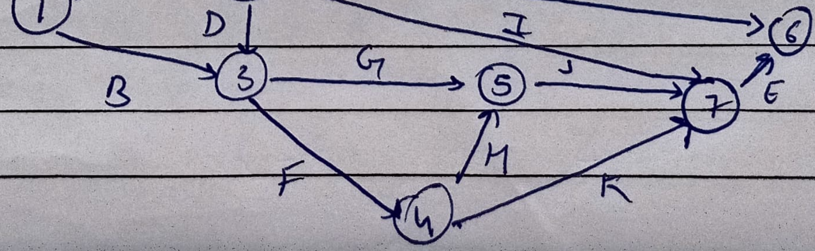


Activity	Time
1-2	4
1-3	1
2-4	1
3-4	1
3-5	6
~~3-5~~	5
4-9	5
5-6	4
5-7	8
6-8	1
7-8	2
8-10	5
9-10	7



A < C, D, I
B < G, F
D < G, F
F < H, K
G, H < J
I, J, K < E



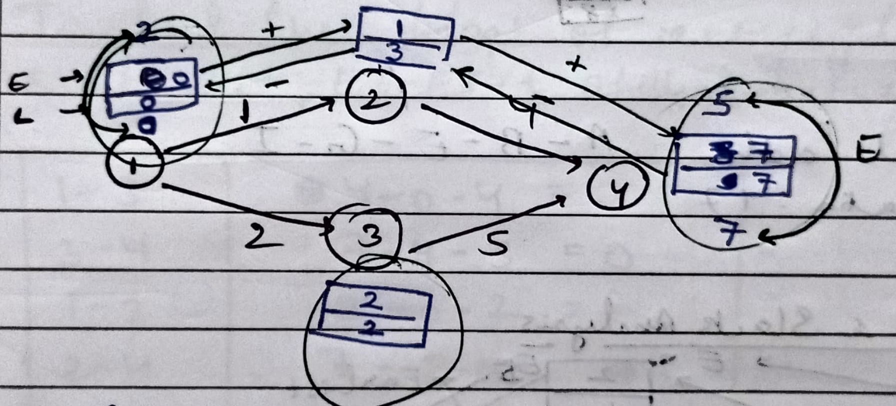


Critical Path Method (CPM)

E_i = earliest occurrence time of event
L_i = latest occurrence time of event
ES = earliest start time of activity
LS = latest start time of activity
EF = Earliest Finish time of activity
LF = latest Finish time of activity
t = duration of activity

- Critical Path is path with maximum duration

Q



if 2 activities are ~~p~~ meeting at a point then in case of E, we take maximum value, and in L, we take minimum

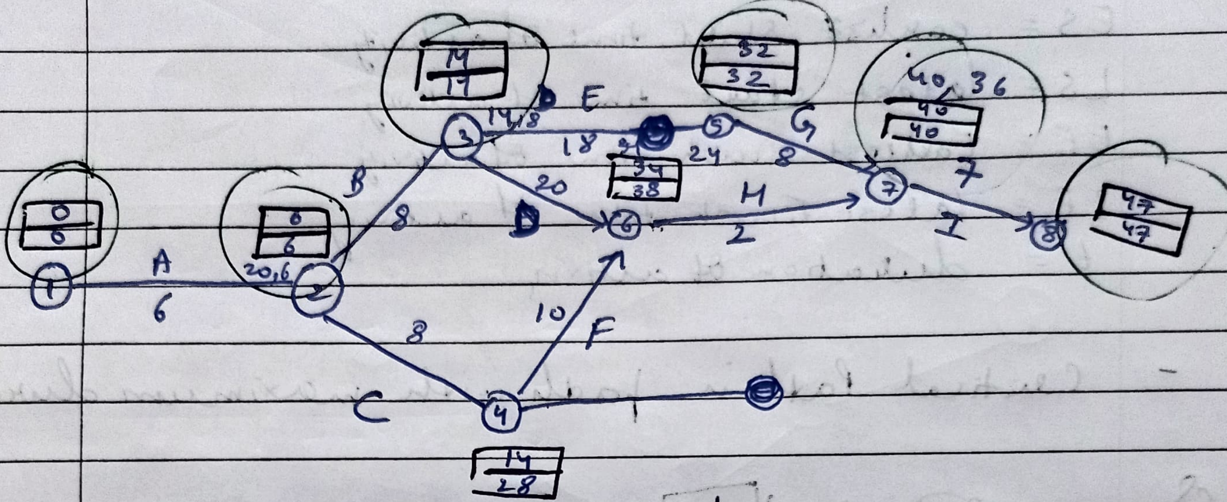
Starting is from 0 and add duration mentioned for each activity

Now check boxes which have both values same it will give path

Critical Path : 1-3-5
Duration : 7

Q

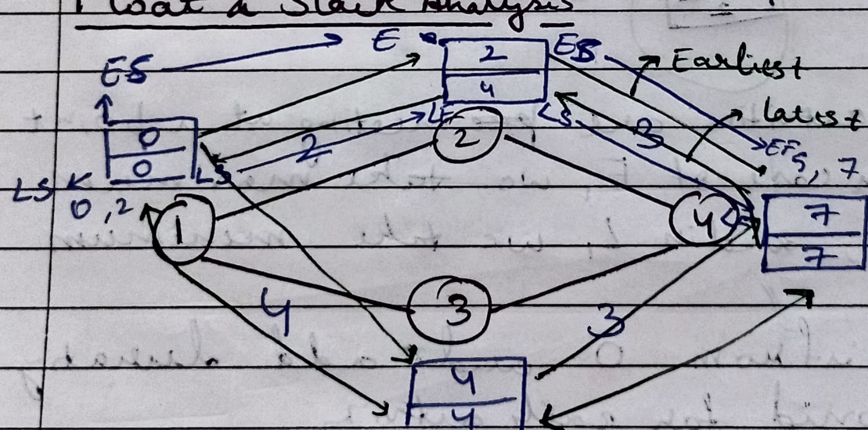
Activities	A	B	C	D	E	F	G	H	I
Predecessor	-	A	A	B	B	C	E	D,F	G,H
Time	6	8	8	20	18	10	8	2	7



Critical Path : A - B - E - G - I

Duration : 47

Float & Slack Analysis

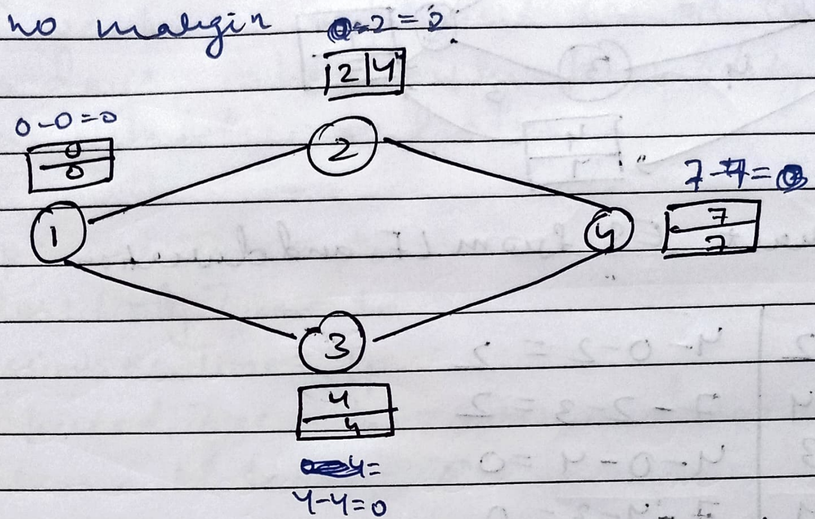


⇒ **Total Float** : Duration to which non-critical activity can be delayed without delaying total project completion time

$$\text{Total Float} = (L_i - E_i) - t = LS - ES = LF - EF$$

__/__/__

To be calculated for non-critical path as critical path takes maximum time so no margin



To find ~~free float of activity from 1 to 3~~ subtract ES-ES-t of that activity

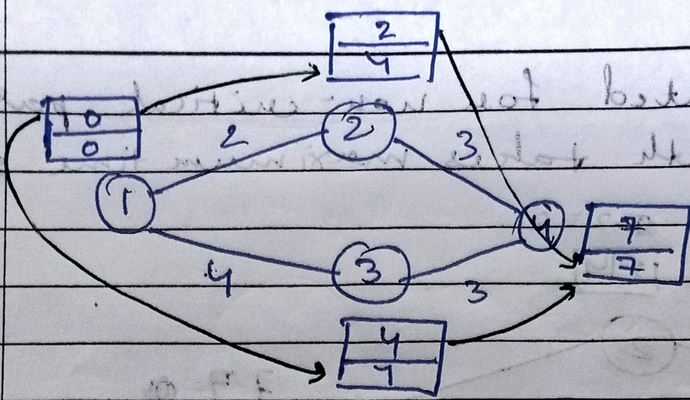
1-3	4-0-4 = 0
3-4	7-4-3 = 0
1-2	2-0-2 = 0
2-4	7-3-2 = 2

⇒ Free Float: Amount of time if an activity is delayed but won't effect start of next activity

Free Float = EF - ES - t

⇒ Independent Float: Amount of starting time delayed of an activity but won't effect start of next activity

Independent Float = LF - ES - t



Subtract ES from LF and duration

1-2 | 4-0-2 = 2
2-4 | 7-2-3 = 2
1-3 | 4-0-4 = 0
3-4 | 7-4-3 = 0

Activity	Time	Earliest Time		Latest Time		TF	FF	IF
		Start	Finish	Start	Finish			
1-2	13	0	13	0	13			
1-3	12	0	12	6	18			
2-4	2	13	15	24	26			
3-4	8	12	20	18	26			
2-5	15	13	28	13	26			
4-5	2	20	22	26	28			

E_i and L_i are for specific event

→ L_i
→ E_i

__/__/__

Project Evaluation and Review Technique

If we know a project can be completed in n days, what is the probability it can complete before and after

- Optimistic Time = t_o
- Most Likely Time = t_m
- Pessimistic Time = t_p
- Expected Time = t_e, same as duration in critical path
- Variance of Activity = σ^2

① Draw network and find expected time.

$$t_e = \frac{1}{6}(t_o + 4t_m + t_p)$$

$$\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$$

② Find critical path

③ Calculate variance of all critical activities and add, as project length is sum of variance of all critical activities

④ Calculate Z to get ~~prop~~ probability of completing task in given time

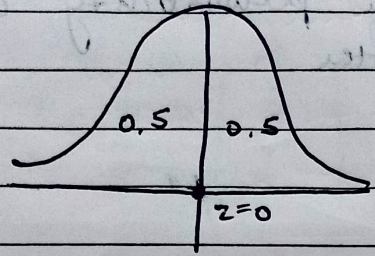
$$Z = \frac{\overbrace{T_s}^{\text{time for which to calculate}} - \underbrace{T_e}_{\text{expected time}}}{\underbrace{\sigma}_{\text{S.d / }\sqrt{\text{variance}}}}$$

~~variance~~

T_s: scheduled time

Z = X.abc

first 2 numbers should be checked vertically in Z table and remaining horizontally



both sides are equal and calculate accordingly and total of each side is 0.5

Crashing Network

- Indirect Cost: Cost related to the whole project rather than a specific activity

Q

Activities	Time (weeks)		Cost (£)	
	Normal	Crash	Normal	Crash
1 - 2	6	3	40,000	70,000
1 - 3	5	3	30,000	52,000
2 - 4	2	1	60,000	87000
3 - 4	10	6	70,000	98,000
2 - 5	3	2	45,000	63000
4 - 5	4	2	26,000	50,000

Indirect Cost £11,500/week

Step 1

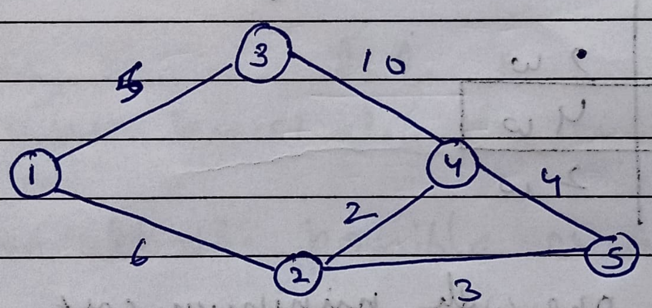
Find cost slope i.e. find the advantage of opting crash ~~time~~ cost by finding difference in normal and crash and dividing it by difference of normal and crash time

$$\text{Cost Slope} = \frac{CC - NC}{NT - CT}$$

Activities	Time		Cost		Cost Slope	Max. Crash Time
	N	C	N	C		
1-2	6	3	40,000	70,000	10,000 /w	3
1-3	5	3	30,000	52,000	11,000/w	2
2-4	2	1	60,000	84,000	24,000/w	1
3-4	10	6	70,000	98000	7,000/w	4
2-5	3	2	45000	63,000	17,000/w	1
4-5	4	2	26000	50,000	12,000/w	2
			2,71,000			

<u>Step-2</u>

Draw network diagram and normal time will be used



<u>Step 3</u>

Find total time for all paths individually and find critical path

1-2-5 : 6+3 = 9

1-2-4-5 : 6+2+4 = 12

1-3-4-5 : 5+10+4 = 19 Highest (critical path)

Time of critical path will be project completion time

Project completion time = 19 weeks

Step 4

Calculate ~~fo~~ total project cost it includes both direct and indirect cost and indirect cost will be based on total project completion time and direct cost is sum of all normal costs.

Project Cost = Direct Cost + Indirect Cost

= 2,71,000 + (11,500 × 19)

= 489500

Step 5

List paths of critical activity

1-3	11,000	2 w
3-4	7,000	4 w
4-5	12,000	2 w

now select the one with minimum cost

Here we can deduct 4w from our critical path but first check with other paths so that our path is not lower than others after deduction

1-2-5 : 9

1-2-4-5 : 12

1-3-4-5 : 19 , if we deduct 4 then also it will be 15

__/__/____

Duration after Crushing			
	I C	II C	III C
1-2-5	9	9	9
1-2-4-5	12	12	10
1-3-4-5	15	13	11

Still critical path is same
now check new interval
with lowest cost

1-3 ⇒ 11,000 (2w)

As deducting 2 from 15 will not go below
next lowest, so we can do

4-5 ⇒ 12,000 (2w)

here we can deduct 2 coz 4-5 is present
in next lowest also, so both will decrease by 2

No all the possible crushing time is exhausted

We stop when all possible crushing is exhausted
or critical becomes equal to or no deduction
possible.

Step-6

we have to plot all the cost and
crushing from the above table

Time Relation Table

Duration ~~of~~ Critical Path	Crashing Activities	Crashing Cost	Total CC (A)	IN C (B)	I C (C)	Total Cost : A+B+C
19w	~~(3-4) by 4w~~ –	–	–	271000	11800 x 19	489500
15w	(3-4) by 4w	(7000 ~~7800~~ x 4)	28000	271000	11500x15	471500
13w	(1-3) by 2w	(11,000 ~~52,000~~ x2)	22,000	271000	11500x13	470500
11w	(4-5) by 2w	(12,000 ~~50,000~~ x2)	24000	271000	11500x11	471500

minimum from cost POV.

Time POV