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# Transportation

⇒ **North West Corner Rule**

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| Factory | $W_1$ (starting) | $W_2$ | $W_3$ | $W_4$ | Factory Capacity |
|---|---|---|---|---|---|
| $F_1$ | 19 ⁵ | 30 | 50 | 10 | 7 ₂ |
| $F_2$ | 70 | 30 | 40 | 60 | 9 |
| $F_3$ | 40 | 8 | 70 | 20 | 8 |
| warehouse Requirement | 5 ₀ | 8 | 7 | 14 | 34 |

$F$ capacity total: 34

Requirement total: 34

Sum should be same otherwise it be an unbalanced problem

write minimum number on 19 as 5 is over write 0 below and 2 is remaining out of 7 after minus 5

now $W_1$ is satisfied so its over

| Factory | $W_1$ | $W_2$ | $W_3$ | |
|---|---|---|---|---|
| $F_1$ | 30 ² | 50 | 10 | 2 ₀ |
| $F_2$ | 30 | 40 | 60 | 9 |
| $F_3$ | 8 | 70 | 20 | 18 |
| | 8 ₆ | 7 | 14 | |

remove row/column which become 0

| | $W_1$ | $W_2$ | $W_3$ | |
|---|---|---|---|---|
| $F_1$ | 30 ⁶ | 40 | 60 | 9 ₃ |
| $F_2$ | 8 | 70 | 20 | 18 |
| | 6 ₀ | 7 | 14 | |

| | $W_1$ | $W_2$ | |
|---|---|---|---|
| $F_1$ | 40 ³ | 60 | 3 ₀ |
| $F_2$ | 70 | 20 | 18 |
| | 7 ₄ | 14 | |

| | $W_1$ | $W_2$ | |
|---|---|---|---|
| $F_1$ | 70 ⁴ | 20 ¹⁴ | 18 ₁₄ |
| | 4 ₀ | 14 | |

in last table put remaining number of the remaining digit now in the base table write numbers received by each value

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | |
|---|---|---|---|---|---|
| $F_1$ | 19 (5) | 30 (2) | 50 | 10 | 7 |
| $F_2$ | 70 | 30 (6) | 40 (3) | 60 | 9 |
| $F_3$ | 40 | 8 | 70 (4) | 20 (14) | 18 |
| | 5 | 8 | 7 | 14 | 34 |

check for m+n-1 allocation

$3+4-1=6$

So, it is basic feasible solution

Total Cost $= 19\times5 + 30\times2 + 30\times6 + 40\times3 + 70\times4 + 20\times14$

$= 85+60+180+120+280+280$

$= 1015$

## Least Cost Method

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | S |
|---|---|---|---|---|---|
| $F_1$ | 19 | 30 | 50 | 10 | 7 |
| $F_2$ | 70 | 30 | 40 | 60 | 9 |
| $F_3$ | 40 | 8 [8] | 70 | 20 | $18^{10}$ ← |
| D | 5 | 8 ↑0 | 7 | 14 | |

circle minimum delivery cost

out of 8 and 18 min is 8

| | $W_1$ | $W_3$ | $W_4$ | |
|---|---|---|---|---|
| $F_1$ | 19 | 50 | $10^{7}$ | $7_0$ |
| $F_2$ | 70 | 40 | 60 | 9 |
| $F_3$ | 40 | 70 | 20 | 10 |
| | 5 | 7 | $14_7$ | |

| | $W_1$ | $W_3$ | $W_4$ | |
|---|---|---|---|---|
| $F_2$ | 70 | 40 | 60 | 9 |
| $F_3$ | 40 | 70 | $20^{4}$ | $10^3$ |
| | 5 | 7 | $7_0$ | |

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| | $W_1$ | $W_3$ | | |
|---|---|---|---|---|
| $F_1$ | 70 | 40 $^{7}$ | $9^{2}$ | |
| $F_3$ | 40 | 70 | 3 | |
| | 5 | $7_0$ | | |

we can allocate on any of the 40

| | $W_1$ | |
|---|---|---|
| $F_1$ | $70^{2}$ | 2 |
| $F_3$ | 40 $^{3}$ | 3 $^{5}$ |
| | $5^{0}$ | |

on last step we give them their correspondy value

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | |
|---|---|---|---|---|---|
| $F_1$ | 19 | 30 | 50 | $10^{7}$ | 7 |
| $F_2$ | $70^{2}$ | 30 | $40^{7}$ | 60 | 9 |
| $F_3$ | $40^{3}$ | $8^{8}$ | 70 | $20^{7}$ | 18 |
| | 5 | 8 | 7 | 14 | |

Total Cost = $70 \times 2$ + $40 \times 3$ + $8 \times 8$ + $40 \times 7$ + $7 \times 10$
+ $20 \times 7$
= 814

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| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | | |
|---|---|---|---|---|---|---|
| $F_1$ | 19 | 30 | 50 | 10 | $^{2}$7 | (9) |
| $F_2$ | 40 | 30 | 40 | 60 | 9 | (10) |
| $F_3$ | 40 | 8 $^{8}$ | 70 | 20 | $^{10}$18 | (12) |
| | 5 | $8_0$ | 7 | 14 | 34 | |
| | (21) | (22) | (10) | (10) | | |

check the lowest value and 2nd lowest in a column and write their diff: then doin row
now check for highest penalty
In the column/row check for minimum
now do just like did in previous method
but in this case we will get new penalty each time

| | $W_1$ | $W_3$ | $W_4$ | | |
|---|---|---|---|---|---|
| $F_1$ | 19 $^{5}$ | 50 | 10 | $7^{2}$ | (9) |
| $F_2$ | 40 | 40 | 60 | 9 | (20) |
| $F_3$ | 40 | 70 | 20 | 10 | (20) |
| | $5_0$ | 7 | 14 | | |
| | (21) | (10) | (10) | | |

| | $W_3$ | $W_4$ | | |
|---|---|---|---|---|
| $F_1$ | 50 | 10 | 2 | (40) |
| $F_2$ | 40 | 60 | 9 | (20) |
| $F_3$ | 70 | $20^{10}$ | $10_{10}$ | (50) |
| | 7 | $14_4$ | | |
| | (10) | (10) | | |

| | $W_3$ | $W_4$ | | |
|---|---|---|---|---|
| $F_1$ | 50 | $10^2$ | 20 | (40) |
| $F_2$ | 40 | 60 | 9 | (20) |
| | 7 | $4^2$ | | |
| | (10) | (50) | | |

| | $W_3$ | $W_4$ | |
|---|---|---|---|
| $F_2$ | $40^7$ | $60^2$ | $19^0$ |
| | $7^0$ | $2^0$ | |

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | |
|---|---|---|---|---|---|
| $F_1$ | $19^5$ | 30 | 50 | $10^2$ | 7 |
| $F_2$ | 40 | 30 | $40^7$ | $60^2$ | 9 |
| $F_3$ | 40 | $8^8$ | 70 | $20^{10}$ | 18 |
| | 5 | 8 | 7 | 14 | |

Total Cost $= 19 \times 5 + 8 \times 8 + 40 \times 7 \quad \times 10 \times 2 \quad \times 60 \times 2 \times 10 \times$

$= 779$

<u>Optimality Test</u>

To check if cost can be further lowered

## Modi Method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| $O_1$ | $1^{20}$ | 2 | $1^{10}$ | 4 | 30 |
| $O_2$ | 3 | $3^{20}$ | $3^{20}$ | $1^{10}$ | 50 |
| $O_3$ | 4 | $2^{20}$ | 5 | 9 | 20 |
| | 20 | 40 | 30 | 10 | |

Total Cost = 180

**For allocated Cells** $c_{ij} = u_i + v_j$

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| $O_1$ | $1^{20}$ | . | $1^{10}$ | . | $u_1 \Rightarrow -2$ |
| ⇒ $O_2$ | . | $3^{20}$ | $3^{20}$ | $1^{10}$ | $u_2 = 0$ |
| $O_3$ | . | $2^{20}$ | . | . | $u_3 \Rightarrow -1$ |
| | $v_1 \Downarrow 3$ | $v_2 \Downarrow 3$ | $v_3 \Downarrow 3$ | $v_4 \Downarrow 1$ | |

those values which have allocation mention them

There sum should be equal to cost

Check which row/column has highest no. of allocation and assume its value 0

put these value here

**For non-allocated Cells** $c_{ij} - (u_i + v_j) \geq 0$

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| $O_1$ | . | $^{+}2^{1}$ | . | $^{+}4^{-1}$ | $u_1\,(-2)$ |
| $O_2$ | $3^{3+}$ | . | . | . | $u_2\,(0)$ |
| $O_3$ | $4^{2+}$ | . | $^{+}5^{2+}$ | $9^{0}$ | $u_3\,(-1)$ |
| | $v_1$ (3) | $v_2$ (3) | $v_3$ (3) | $v_4$ (1) | |

As $c_{ij} - (u_i + v_j) \geq 0$

So, ~~the~~ cost 180 is optimal

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Q when non-allocated becomes negative

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | |
|---|---|---|---|---|---|
| $F_1$ | | $30^{-2}$ | $50^{10}$ | | $u_1 = 10$ |
| $F_2$ | $70^{69}$ | $30^{48}$ | | | $u_2 = 60$ |
| $F_3$ | $40^{29}$ | | $70^{0}$ | | $u_3 = 20$ |
| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | |
| | 9 | -12 | -20 | 0 | |

at (2,2) cell $c_{22} - (u_2 + v_2) \leq 0$

Now allocate $\theta$ at (2,2) cell
it will done in the table of allocated

| | | | |
|---|---|---|---|
| $19^{5}$ | | $10^{2}$ | 7 |
| | $30^{\theta} \leftarrow 40^{7}$ | $60^{2-\theta}$ | $9+\cancel{\theta}$ |
| | $8^{8+\theta} \rightarrow$ | $\uparrow 20^{10+\theta}$ | $18-\cancel{\theta}$ |
| 5 | $8+\cancel{\theta}$ 7 | $14+\cancel{\theta}$ | |

check above and below $30^{\theta}$ for remove $\theta$ from $8+\theta$. then go to the  [illegible]

wherever $\theta$ takes a new position outside constant will have effect

Now of all the values of the edges take the -ve for minimizing

$$\min(8-\theta, 2-\theta) = 0$$

$8 = \theta$ $\quad$ $2 = \theta \Leftarrow$ minimum

so $\theta = 2$

Now find cost by putting value of $\theta$ in allocated table.

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## Unbalanced

Check the sum of Demand and supply whichever is less give it a dummy row/column

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| | 4 | 8 | 8 | 76 |
| | 16 | 24 | 16 | 82 |
| | 8 | 16 | 24 | 77 |
| | 72 | 102 | 41 | |

diff = 20

Give dummy column all values 0 and outside value is excess value

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| | 4 | 8 | 8 | 0 | 76 |
| | 16 | 24 | 16 | 0 | 82 |
| | 8 | 16 | 24 | 0 | 77 |
| | 72 | 102 | 41 | 20 | |

Now solve normally

~~Prohibited Routes~~

## Maximization

Whenever it is mentioned transportation cost minimize it and when its is profit maximize it

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Table give for max can't be done we have to convert it for min

For conversion take highest cost and write it's difference with other costs

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $S_1$ | 8 | 9 | 6 | 120 |
| $S_2$ | 5 | 10 | 12 | 80 |
| $S_3$ | 3 | 9 | 10 | 80 |
| | 150 | 80 | 50 | |

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $S_1$ | 7 | 10 | 9 | 120 |
| $S_2$ | 0 | 5 | 3 | 80 |
| $S_3$ | 12 | 6 | 5 | 80 |
| | 150 | 80 | 50 | |

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $S_1$ | $7^{70}$ | $10^{50}$ | 9 | 120 |
| $S_2$ | $0^{80}$ | 5 | 3 | 80 |
| $S_3$ | 12 | $6^{30}$ | $5^{50}$ | 80 |
| | 150 | 80 | 50 | |

now revert back to original table with allocate at pos.

| | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
| $S_1$ | $8^{70}$ | $5^{50}$ | 6 |
| $S_2$ | $15^{80}$ | 10 | $12^{50}$ |
| $S_3$ | 3 | $9^{30}$ | 10 |

Now just multiply.

## Alternate Solution

When checking under Modi, if in second phase difference of any cost i.e. $c_{ij} - (u_i + v_j)$ is 0 alternate exist

| | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
| $S_1$ | $+6^{25}$ | $3^{25}-$ | 5 |
| $S_2$ | $-5$ | $2^{40}+$ | $2^{40}$ |
| $S_3$ | 12 | $7^{85}$ | 8 |

The cell for which we get zero is the starting point, starting pt. will be +ve then next will be -ve

We will take change to be -ve corner of loop

Change = 3 (as 3 < 5)

Now add change where +ve sign and subtract where -ve sign

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $S_1$ | $9^{35}$ | $0^{25}$ | 5 | 65 |
| $S_2$ | 3 | $5^{40}$ | $2^{40}$ | 0 |
| $S_3$ | 12 | $7^{85}$ | 8 | 20 |
| | 15 | 65 | 40 | |

take $u_1 = 0$ for row having maximum allocation