Game Theory

Player: Any person in a game

Strategy: course of action available based on advanced knowledge

Pure

when same strategy used each time

Mixed

selects his course of action in accordance with some fixed probability

Optimum

which puts a player in most preferred position

2 Person Zero Sum Game

Only 2 players play and gain of one is loss of other in same amount

Fair Game: value of game is 0

Solution of Game having Saddle Point (Pure Strategy)

If max-min value equals min-max value the game is said to have a saddle point and corresponding strategy is optimum

Q2 Solve 2 person 0 sum game with 3x2 pay off matrix of player A

Player B

$$
\begin{array}{c|cc|c}
\text{Player A} & B_1 & B_2 & \text{R-min} \\
\hline
A_1 & 9 & 2 & 2 \\
\Rightarrow A_2 & 8 & 6 & 6 \\
A_3 & 6 & 4 & 4 \\
\hline
\text{C-Max} & 9 & 6 &
\end{array}
$$

Max-Min
↓
Max value in R-min

Min-Max
↓
Min value in C-Max

As both are equal there is saddle point

Optimal : A is A_2

B is B_2

Value of game = saddle pt. = 6

Game without Saddle Pt. (Mixed Strategy)

It can be solved by following Methods:

⇒ Algebric

⇒ Graphical Method

⇒ LPP Method

Dominance Rule

R-min C-Max

① Row Dominance

$$\begin{array}{cc|ccc} & & C_1 & C_2 & C_3 \\ A & R_1 & 4 & 6 & -5 \\ & R_2 & 5 & 6 & -3 \end{array}$$

$4 < 5$
$6 = 6$
$-5 < -3$

② Column Dominance

$$\begin{array}{cc|cc} & & C_1 & C_2 \\ & R_1 & 4 & 2 \\ A & R_2 & 0 & -1 \\ & R_3 & -3 & -3 \end{array}$$

$4 > 2$
$0 > -1$
$-3 = -3$

In row dominance we compare row and delete min one and in column we delete ~~min~~ max column

Row Modified

when we can't delete row

$$\begin{array}{cc|cc} & & B_1 & B_2 \\ & A_1 & 2 & 4 \\ A & A_2 & 5 & 0 \\ & A_3 & 0 & 8 \end{array}$$

3.5 2

for this combination not possible

2.5 4 this is ok

$3.5 > 0$
$2 < 8$

we are not able to delete so take average of any 2 rows (and compare) with 3^{rd}

Column Modified

8

3.5

$$\begin{array}{cc|ccc} & & B_1 & B_2 & B_3 \\ A & A_1 & 8 & 15 & 1 \\ & A_2 & 3 & -1 & 4 \end{array}$$

11 1

0 4

not possible

Q

$$\begin{array}{cc|ccc} & & B_1 & B_2 & B_3 \\ & A_1 & 4 & 5 & 8 \\ A & A_2 & 6 & 4 & 6 \\ & A_3 & 4 & 8 & 4 \end{array}$$

—/—/—

①
$$\begin{Bmatrix} 4 & 5 & 8 \\ 6 & 4 & 6 \\ \cancel{4} & \cancel{8} & \cancel{4} \end{Bmatrix}$$

②
$$\begin{bmatrix} 4 & 5 & \cancel{8} \\ 6 & 4 & \cancel{6} \end{bmatrix}$$

$$\begin{array}{c|cc|} & B_1 & B_2 \\ A_1 & 4 & 5 \\ A_2 & 6 & 4 \end{array}$$

$2 \quad \frac{2}{2+1} = \frac{2}{3}$

$1 \quad \frac{1}{2+1} = \frac{1}{3}$

always if getting negative num (−) positive

Optimal strategy of A is $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$ → as A_3 was deleted

$$\begin{array}{ccc} & B_1 & B_2 \\ A_1 & 4 & 5 \\ A_2 & 6 & 4 \\ & 1 & 2 \\ & \frac{1}{3} & \frac{2}{3} \end{array}$$

Optimal strategy of B is $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$

Value of game $= \frac{2}{3} \times 4 + \frac{1}{3} \times 6$

$= \frac{8}{3} + \frac{6}{3} = \frac{14}{3}$

Graphical Method

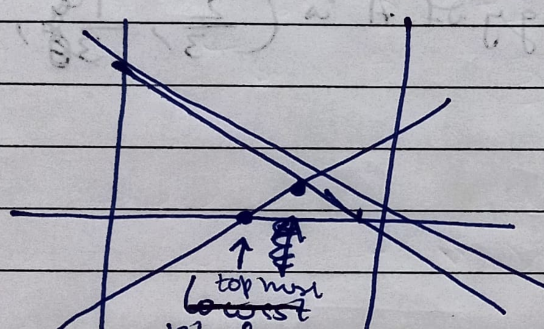
useful for $2 \times M$ or $N \times 2$ order

Q Solve following 2×3 by graph method

$$
\begin{array}{cc} & B \\ & \begin{array}{ccc} I & II & III \end{array} \\ A \begin{array}{c} I \\ II \end{array} & \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix} \end{array}
$$

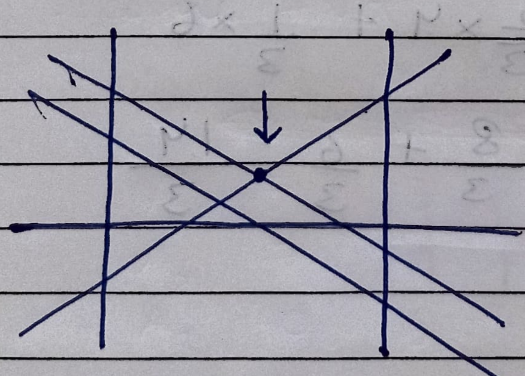
All forms have to be converted to 2×2

If problem is 2×3 then graph will be drawn



↑ top most ~~lowest~~ pt from below

when 3×2 then

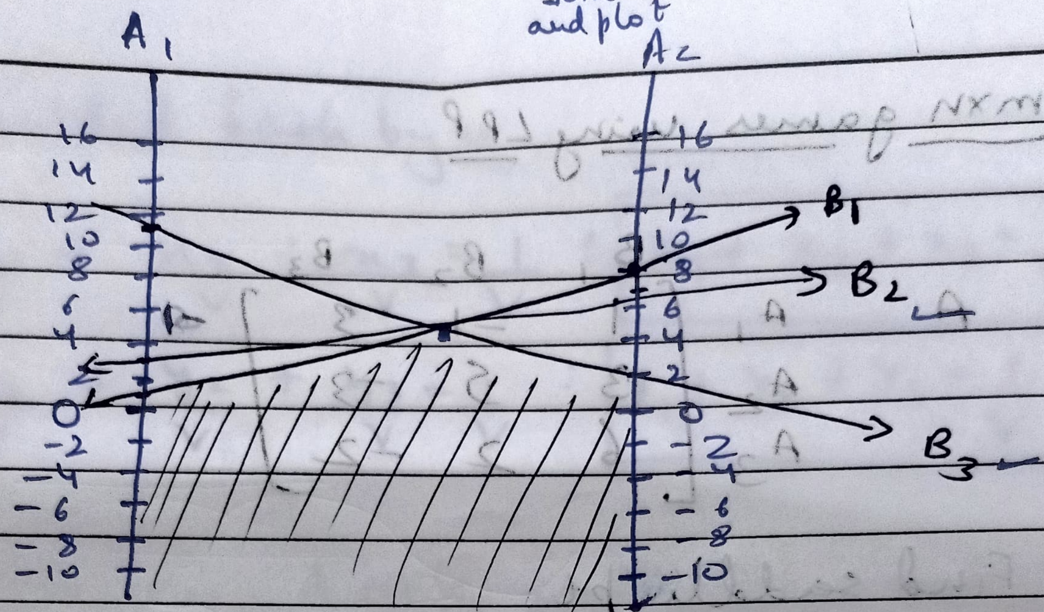


Player have 2 moves/column will be taken on graph

here A

$$A\quad \begin{array}{c|ccc} & I & B\ II & III \\ \hline I & 1 & 3 & 11 \\ II & 8 & 5 & 2 \end{array}$$

Join and plot



the highest pt. from below falling on the two lines with b_B take i.e B_2 and B_3

$$A\quad \begin{array}{c|cc|c|c} & II & III & & \\ \hline I & 3 & 11 & 3 & 3/11 \\ II & 5 & 2 & 8 & 8/11 \\ \hline & 9 & 2 & & \\ & 9/11 & 2/11 & & \end{array}$$

Optimal strat of A $\left(\frac{3}{11}, \frac{8}{11}\right)$

B $\left(0, \frac{9}{11}, \frac{2}{11}\right)$

$$v = \frac{9}{11} + \frac{40}{11} = \frac{49}{11}$$

mxn games using LPP

Q

$$A \quad \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 1 & -1 & 3 \\ A_2 & 3 & 5 & -3 \\ A_3 & 6 & 2 & -2 \end{array}$$

① Find saddle pt.

$$\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 3 & 5 & -3 & -3 \\ 6 & 2 & -2 & -2 \\ \hline 6 & 5 & 3 & \end{array}$$

No saddle pt. but value of game lie b/w $-1 \leq v \leq 3$

② here maximum ~~val~~ -ve value is -3 among all the values, so we add constant $K=4$ so all values become +ve

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \\ \hline A_1 & 5 & 3 & 7 & x_1 \\ A_2 & 7 & 9 & 1 & x_2 \\ A_3 & 10 & 6 & 2 & x_3 \\ \hline & y_1 & y_2 & y_3 & \end{array}$$

Let A be x_1, x_2, x_3 and B be y_1, y_2, y_3

$x_1 + x_2 + x_3 = 1$ coz A can have any of the strategy

$y_1 + y_2 + y_3 = 1$

divided both by v

(3) $\frac{x_1}{V} + \frac{x_2}{V} + \frac{x_3}{V} = \frac{1}{V}$: $\boxed{X_1 + X_2 + X_3 = \frac{1}{V}}$

$\frac{y_1}{V} + \frac{y_2}{V} + \frac{y_3}{V} = \frac{1}{V}$: $\boxed{Y_1 + Y_2 + Y_3 = \frac{1}{V}}$

we assume A to be gaining player and B to be ~~losing~~ minimizing

B want min V, so $\frac{1}{V}$ = max

~~A want to maxV~~

LPP: Max $\frac{1}{V} = Y_1 + Y_2 + Y_3$

$3Y_1 - 2Y_2 + 4Y_3 \leq 1$

$-1Y_1 + 4Y_2 + 2Y_3 \leq 1$

$2Y_1 + 2Y_2 + 6Y_3 \leq 1$

solve like LPP

As the LPP is in terms of Y we will get it's value

for value of X we have to see the value of slack variables in $c_j - z_j$ column and equate them in X_1, X_2, X_3