Simplex Method

- used for maximizing situation

Q $\text{Max } Z = 5x_1 + 3x_2$

$3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$

① Add slack variable (s_1, s_2) or any other

$3x_1 + 5x_2 + S_1 + 0S_2 = 15$

$5x_1 + 2x_2 + S_2 + 0S_1 = 10$

$Z = 5x_1 + 3x_2 + 0S_1 + 0S_2$

② Make table.

Basis		x_1	x_2	s_1	s_2	b_i
S_1	0	3	5	1	0	15
S_2	0	5	2	0	1	10
C_j		5	3	0	0	⇐ Z
Z_j		0	0	0	0	

variables Creating unit matrix

Z_j: $x_1 \Rightarrow 0\times3 + 0\times5 = 0$, $x_2 \Rightarrow 0\times5 + 0\times2 = 0$, $S_1 = 0$, $S_2 = 0$

Now we want to remove S_1 and S_2 from Basis by making main variables unit matrix i.e. X_1 & X_2

③ Find Z_j

Multiply basis value with variable value and vertically for that column

(4) Now obtain $c_j - z_j$,

$c_j - z_j$ 5   3 0 0

-ve m_r not considered

Basis		x_1	x_2	s_1	s_2	b_i	m
s_1	0	3	5	1	0	15	$15/3 = 5$
s_2	0	5	2	0	1	10	$10/5 = 2$
c_j		5	3	0	0		
z_j		0	0	0	0		
$c_j - z_j$		5	3	0	0		

(5) Now check $c_j - z_j$ for ~~most~~ highest positive and find minimum ratio

Take ratio of value of column b_i with the values of column having highest $c_j - z_j$ to find minimum ratio

So the slack variable ~~here~~ under basis column having lowest minimum ratio will be replaced with the variable having highest $c_j - z_j$ value in this case s_2 will be replaced by x_1

Now observe the common value of x_1 column and s_2 is 5, so we have to make $5 \Rightarrow 1$ and $3 \Rightarrow 0$ in x_1 column. This can be done by transformation of matrix

(4) Transforming

Basis		x_1	x_2	s_1	s_2	b_i	m
s_1	0	0	19/5	1	-3/5	9	$\frac{9\times5}{19} = \frac{45}{19} = 2.3$
x_1	5	1	2/5	0	1/5	2	$\frac{2\times5}{2} = \frac{10}{2} = 5$
c_j		5	3	0	0		
z_j		5	2	0	1		
$c_j - z_j$		0	1	0	-1		

value of x_1 in Z

Now divide all the variables in line of 5 to make it 1

To make 3, 0 multiply values of x_1 by 3 and subtract from the above row

c_j is constant but find new z_j and follow the process again

So, S_1 will be replaced by x_2 and 19/5 is to be made 1 and 2/5 to be 0'

Basis		x_1	x_2	s_1	s_2	b_i
x_2	3	0	1	5/19	.57	45/19
x_1	5	1	0	-2/19	23	20/19
c_j		5	3	0	0	
z_j		5	3	5/19	.56	
$c_j - z_j$		0	0	-5/19	-56	

$2 - \frac{45}{19} \times \frac{2}{5}$

$\frac{38 - 18}{19} = \frac{20}{19}$

As no $c_j - z_j$ is positive so we have no entering variable for next round and b_i gives the value of variable under basis column to be put in Z

$$z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right)$$

$$= \frac{100}{19} + \frac{135}{19}$$

$$= \frac{235}{19}$$

Minimization Problem

⇒ Either use Big-M method or Two Phase method.

⇒ ~~Only Max problem can be solved~~

Big-M Method

Q. $\text{Min } z = x_1 + x_2$

$2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$, $x_1 \geq 0$

(1) Convert to Max by multiplying by −ve

$$\text{Max } z' = -x_1 - x_2$$

(2) Check the condition eqn and if unit matrix is not forming so with slack variables, artificial variables will also be added

as it was greater than problem slack variable were subtracted due to which unit matrix was not ~~forming so additional~~ variables were added

$$2x_1 + x_2 \underbrace{- S_1 + 0S_2}_{\text{slack}} + \underbrace{A_1 + 0A_2}_{\text{Artificial}} = 4$$

$$x_1 + 7x_2 - S_2 + 0S_1 + A_2 + 0A_1 = 7$$

③ Now optimize max eqⁿ also

$$Max\ Z' = -x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

here it is -ve coz we have transformed eqⁿ from min to max have we solved it in min then it would have been positive

Basis	x_1	x_2	S_1	S_2	A_1	A_2	b_i	m_r
A_1 -M	2	1	-1	0	1	0	4	$4/1 = 4$
A_2 -M	1	7	0	-1	0	1	7	$7/7 = 1$
C_j	-1	-1	0	0	-M	-M		
Z_j	-3m	-8m	m	m	-m	-m		
$C_j - Z_j$	-1+3m	-1+8m	-m	-m	0	0		

now do it like simplex method

④ Now solve like simplex till new table; just 1 exception the variable which is getting replace will lose its column in table

Basis	x_1	x_2	S_1	S_2	A_1	b_i	m_r
A_1 -m	2	1	-1	0	1	4	2
x_2 -1	1/7	1	0	-1/7	0	1	7
C_j	-1	-1	0	0	-m		
Z_j	$-2m - 1/7$	-m - 1	m	1/7	-m		
$C_j - Z_j$	$2m - 6/7$	m	-m	-1/7	0		

Basis	x_1	x_2	s_1	s_2	b_i
x_1 -1	0	-13	-1	2	28
x_2 -1	1/7	1	0	-1/7	1
c_j	-1	-1	0	0	
z_j	-1/7	12	1	15/7	
$c_j - z_j$	-6/7	-13	-1	-15/7	

As no value in $c_j - z_j$ is +ve so we got results $x_1 = 28$ put it in ~~max~~ min Z

$x_2 = 1$

min Z = 124

we can put in Max Z also result will be same as -ve will be remove when max ⇒ min

Two Phase Method

here we won't add artificial variable in condition eq^n

Q ~~Phase~~ Min $Z = x_1 + x_2 \Rightarrow$ Max $Z = -x_1 - x_2$

$2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$

Phase-1

$2x_1 + x_2 - S_1 + 0S_2 + A_1 + 0A_2 = 4$

$x_1 + 7x_2 + 0S_1 - S_2 + 0A_1 + A_2 = 7$

$Z = x_1 + x_2 + 0S_1 + 0S_2 + mA_1 + mA_2$

① put every ~~artificial~~ slack and basic variable as 0 and artificial with m = 1 in case of max ~~and~~ ~~m=1 in case of min~~ m = -1 in case of min

~~$Z = A_1 + A_2$~~

$Z = -A_1 - A_2$

Two Phase Method

$$\text{Min } z = \frac{15}{2}x_1 - 3x_2 + 0x_3$$

$$3x_1 - x_2 - x_3 \geq 3 \qquad x_1 - x_2 + x_3 \geq 2$$

Phase I

$$\text{Max } Z' = -\frac{15}{2}x_1 + 3x_2 + 0x_3 + 0S_1 + 0S_2 - mA_1 - mA_2 \quad \cancel{+A_1 + 0A_2}$$

$$3x_1 - x_2 - x_3 - S_1 + 0S_2 + A_1 + 0A_2 = 3$$

$$x_1 - x_2 + x_3 + 0S_1 - S_2 + 0A_1 + A_2 = 2$$

Take Z'

put $m = 1$ and rest 0

$$\text{Max } Z' = -A_1 - A_2$$

Base		x_1	x_2	x_3	S_1	S_2	A_1	A_2	b_i	m_r
A_1	-1	3	-1	-1	-1	0	1	0	3	1
A_2	-1	1	-1	1	0	-1	0	1	2	2
c_j		0	0	0	0	0	-1	-1		
z_j		-4	2	0	1	1	-1	-1		
$c_j - z_j$		4	-2	0	-1	-1	0	0		
x_1	0	1	$-1/3$	$-1/3$	$-1/3$	0	$\cancel{1/3}$	$\cancel{0}$	1	
A_2	-1	0	$-2/3$	$4/3$	$1/3$	-1	[illegible]	$\cancel{1}$		1
z_j		0	$2/3$	$-4/3$	$-1/3$	1	$\cancel{-2/3}$	$\cancel{-1}$		
$c_j - z_j$		0	$-2/3$	$4/3$	$1/3$	-1	$\cancel{-5/3}$	[illegible]		

$0 + \left(-\frac{3}{4} \times \frac{1}{3}\right)$ $1 + \left(\frac{3}{4} \times \frac{1}{3}\right)$

$1 + \frac{1}{4}$

Base		x_1	x_2	x_3	S_1	S_2	b_i
x_1	0	1	$-1/2$	0	$-1/4$	$-1/4$	$5/4$
x_3	0	0	$-1/2$	1	$1/4$	$-3/4$	$3/4$
c_j		0	0	0	0	0	
z_j		0	0	0	0	0	

Phase-I over :-

<u>Phase-II</u> :-

$Z' = -\frac{15}{2}x_1 + 3x_2 + 0x_3 + 0s_1 + 0s_2$

no use of ~~altern~~ artificial variable

copy the above table

Base		x_1	x_2	x_3	S_1	S_2	b_i
x_1	$-15/2$	1	$-1/2$	0	$-1/4$	$-1/4$	$5/4$
x_3	0	0	$-1/2$	1	$1/4$	$-3/4$	$3/4$
z_j		$-15/2$	$15/4$	0	$15/8$	$15/8$	
c_j		$-15/2$	3	0	0	0	
$c_j - z_j$		0	$-3/4$	0	$-15/8$	$-15/8$	

no more positive

So answer is $x_1 = 5/4$, $x_3 = 3/4$

Cases

⇒ **Mixed Constraints**

'≤' : Add only slack

'≥' : Subtract slack and add alternative

'=' : add only alternative
when checking for Basis column variable
A will supersed S

⇒ **Infeasible**

If artificial variable is still there in final table

⇒ **Multiple**

When non-basic variables have value 0, even any single of them then multiple solution exists (in final $c_j - z_j$)

⇒ **Unbounded**

When looking for minimum ratio if all ratios are either -ve or ∞, then it is unbounded